

Materials discussed on 22/11

1. Holomorphic map preserves angles at z_0 where $f'(z_0) \neq 0$.
2. Montel's theorem states that a family \mathcal{F} of holomorphic functions in open connected set G is normal if and only if it is locally bounded. Use the theorem to show the following.

- (a) If $\{f_n\}$ is locally bounded, and

$$\{z \in G : \lim_{n \rightarrow \infty} f_n = f\}$$

has a limit point for some $f \in H(G)$, then $f_n \rightarrow f$.

- (b) Show that the following families are normal.

i.

$$F = \{f \in H(G) : \int_G |f|^2 dx dy \leq M\}$$

ii.

$$F = \{f \in H(G) : f(z_0) = w_0, \operatorname{Re}(f) > 0, \forall z \in G\}$$

where $z_0 \in G$, $w_0 \in \mathbb{C}$, $\operatorname{Re}(w_0) > 0$.